CS103
Winter 2025

Lecture 03:

Propositional Logic

Logistics: Lecture Participation

Lecture Participation

- Starting Wednesday, we will be using the website PollEV to ask questions in lecture for attendance credit.
- If you answer these questions in lecture, you'll get attendance credit for the day.
  - You don't need to have the right answers – you just need to respond to the questions.
- CGOE students: We automatically opt you out of participation, since we assume you aren't physically here.
- If you'd prefer not to attend lectures, that's okay! You can opt to count your final exam in place of participation.
  - We'll send out a form where you can opt-out of participation in Week 4.

Do not miss this deadline!

Lecture Participation

- We'll dry-run PollEV questions today.
- Let's start with the following warm-up:

Make a music recommendation!

Answer at
https://cs103.stanford.edu/pollev

Click "**Register**" and enter your **Stanford e-mail** to get to the SUNet login page.

Also:

pollev.com/**103**

- Here are a few music recs of our own:
  - Jami Sieber - *Timeless*.
  - Aaron Parks - *Little Big* and *Little Big II*.
  - Arthur Moon - NPR Music Tiny Desk Concert.
  - Shakey Graves – *Roll the Bones* (check out *Audiotree Live* version).

Propositional Logic

Question: How do we formalize the definitions and reasoning we use in our proofs?

Where We're Going

- ***Propositional Logic*** (Today)
  - Reasoning about Boolean values.
- ***First-Order Logic*** (Wednesday/Friday)
  - Reasoning about properties of multiple objects.

Outline for Today

- ***Propositional Variables***
  - Booleans, math edition!
- ***Propositional Connectives***
  - Linking things together.
- ***Truth Tables***
  - Rigorously defining connectives.
- ***Simplifying Negations***
  - Mechanically computing negations.

Propositional Logic

$$TakeMath51 \lor TakeCME100$$

$$\neg FirstSucceed \to TryAgain$$

$$IsCardinal \land IsWhite$$

$$TakeMath51 \lor TakeCME100$$

$$\neg FirstSucceed \to TryAgain$$

$$IsCardinal \land IsWhite$$

$$TakeMath51 \lor TakeCME100$$

$$\neg FirstSucceed \to TryAgain$$

$$IsCardinal \land IsWhite$$

These are ***propositional variables***. Each propositional variable stands for a ***proposition***, something that is either true or false.

$$TakeMath51 \lor TakeCME100$$

$$\neg FirstSucceed \to TryAgain$$

$$IsCardinal \land IsWhite$$

These are ***propositional connectives***, which link propositions into larger propositions

Propositional Variables

- In propositional logic, individual propositions are represented by ***propositional variables***.

- Each variable can take one one of two values: true or false. You can think of them as **`bool`** values.

Propositional Connectives

- There are seven propositional connectives, five of which will be familiar from programming.
- First, there's the logical "NOT" operation:

$$\neg p$$

- You'd read this out loud as "not *p*."
- The fancy name for this operation is ***logical negation***.

Truth Tables

- A ***truth table*** is a table showing the truth value of a propositional logic formula as a function of its inputs.
- Let’s examine the truth tables for the connectives we’re exploring today!

“I don’t love cupcakes.”

“I don’t love cupcakes.”

LoveCupcakes : I love cupcakes.

“I don’t love cupcakes.”

LoveCupcakes : I love cupcakes.

$\neg$***LoveCupcakes***

Propositional Variables

- In propositional logic, individual propositions are represented by ***propositional variables***.
- Each variable can take one one of two values: true or false. You can think of them as **`bool`** values.
- In a move that contravenes programming style conventions, propositional variables are usually represented as lower-case letters, such as p, q, r, s, etc.
  - That said, there's nothing stopping you from using multiletter names!

“I don’t love cupcakes.”

LoveCupcakes : I love cupcakes.

$\neg$***LoveCupcakes***

"I don't love cupcakes."

$\boldsymbol{c}$: I love cupcakes.

$\neg \boldsymbol{LoveCupcakes}$

"I don't love cupcakes."

$\boldsymbol{c}$: I love cupcakes.

$$\neg \boldsymbol{c}$$

Propositional Connectives

- There are seven propositional connectives, five of which will be familiar from programming.
- Next, there's the logical "AND" operation:

$$\boldsymbol{p \land q}$$

- You'd read this out loud as "p and q."
- The fancy name for this operation is ***logical conjunction***.

“It’s cardinal and white.”

"It's cardinal and white."

IsCardinal : It's cardinal.

“It’s cardinal and white.”

IsCardinal : It’s cardinal.

IsWhite : It’s white.

“It’s cardinal and white.”

IsCardinal : It’s cardinal.

IsWhite : It’s white.

IsCardinal ∧ ***IsWhite***

“It’s cardinal and white.”

$\boldsymbol{p}$: It’s cardinal.

$\boldsymbol{q}$: It’s white.

$\boldsymbol{IsCardinal} \wedge \boldsymbol{IsWhite}$

"It's cardinal and white."

$\boldsymbol{p}$: It's cardinal.

$\boldsymbol{q}$: It's white.

$$\mathbf{p} \wedge \boldsymbol{q}$$

Propositional Connectives

- There are seven propositional connectives, five of which will be familiar from programming.
- Then, there's the logical "OR" operation:

$$\boldsymbol{p \lor q}$$

- You'd read this out loud as "*p* or *q*."
- The fancy name for this operation is ***logical disjunction***. This is an *inclusive* or.

“You must take Math 51 or CME 100.”

“You must take Math 51 or CME 100.”

TakeMath51 : You must take Math 51.

"You must take Math 51 or CME 100."

TakeMath51 : You must take Math 51.

TakeCME100 : You must take CME 100.

“You must take Math 51 or CME 100.”

$\textit{\textbf{TakeMath51}}$: You must take Math 51.

$\textit{\textbf{TakeCME100}}$: You must take CME 100.

$$\textit{\textbf{TakeMath51}} \lor \textit{\textbf{TakeCME100}}$$

"You must take Math 51 or CME 100."

TakeMath51 : You must take Math 51.

TakeCME100 : You must take CME 100.

TakeMath51 $\lor$ ***TakeCME100***

These are ***propositional variables***. Each propositional variable stands for a ***proposition***, something that is either true or false.

“You must take Math 51 or CME 100.”

TakeMath51 : You must take Math 51.

TakeCME100 : You must take CME 100.

$\textbf{\textit{TakeMath51}} \lor \textbf{\textit{TakeCME100}}$

This is a ***propositional connective***, which links propositions into larger propositions

"You must take Math 51 or CME 100."

$\boldsymbol{TakeMath51}$: You must take Math 51.

$\boldsymbol{TakeCME100}$: You must take CME 100.

$$\boldsymbol{TakeMath51} \lor \boldsymbol{TakeCME100}$$

“You must take Math 51 or CME 100.”

$\boldsymbol{p}$: You must take Math 51.

$\boldsymbol{q}$: You must take CME 100.

$\boldsymbol{TakeMath51} \lor \boldsymbol{TakeCME100}$

"You must take Math 51 or CME 100."

$\boldsymbol{p}$: You must take Math 51.

$\boldsymbol{q}$: You must take CME 100.

$$\boldsymbol{p} \lor \boldsymbol{q}$$

Propositional Connectives

- There are seven propositional connectives, five of which will be familiar from programming.
- There's also the "truth" connective:

$$\top$$

- You'd read this out loud as "true."
- Although this is technically considered a connective, it "connects" zero things and behaves like a variable that's always true.

Propositional Connectives

- There are seven propositional connectives, five of which will be familiar from programming.
- Finally, there's the "false" connective.

$$\bot$$

- You'd read this out loud as "false."
- Like $\top$, this is technically a connective, but acts like a variable that's always false.

Inclusive and Exclusive OR

- The ∨ connective is an *inclusive* “or.” It's true if at least one of the operands is true.
  - It’s similar to the `||` operator in C, C++, Java, etc. and the `or` operator in Python.
- Sometimes we need an *exclusive* “or,” which isn’t true if both inputs are true.
- We can build this out of what we already have.

Write a propositional logic formula for the exclusive OR of p and q.

Answer at
https://cs103.stanford.edu/pollev

Quick Question:

What would I have to show you to convince you that the statement $p \wedge q$ is false?

Quick Question:

What would I have to show you to convince you that the statement $p \lor q$ is false?

de Morgan's Laws

$\neg(p \wedge q)$ ***is equivalent to*** $\neg p \vee \neg q$

$\neg(p \vee q)$ ***is equivalent to*** $\neg p \wedge \neg q$

de Morgan’s Laws in Code

- ***Pro tip:*** Don't write this:

```
if (!(p() && q())) {
    /* … */
}
```

- Write this instead:

```
if (!p() || !q()) {
    /* … */
}
```

- (This even short-circuits correctly: if `p()` returns false, `q()` is never evaluated.)

Mathematical Implication

Implication

- We can represent implications using this connective:

$$\boldsymbol{p \to q}$$

- You'd read this out loud as "*p* implies *q*."
  - The fancy name for this is the ***material conditional***.
- ***Question:*** What should the truth table for $p \to q$ look like?

p	q	$p \to q$
F	F	___
F	T	___
T	F	___
T	T	___

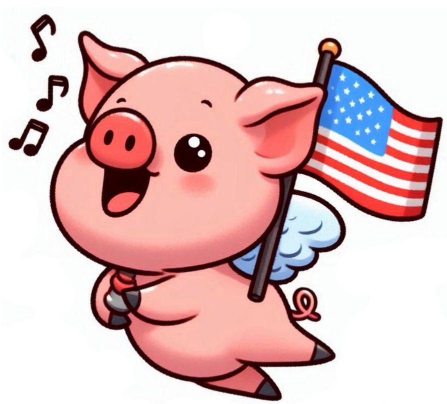
How should we fill in these blanks?

Answer at **https://cs103.stanford.edu/pollev**

The ***pig*** does the thing.

Sean throws ***qookies***.

Contract upheld?



p	q	$p \to q$

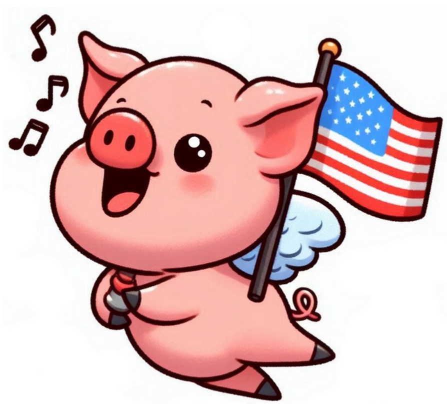
A Contract (from Friday):

If a flying pig bursts into the room and sings a pitch-perfect version of the national anthem, then Sean will throw cookies to the class.

The ***pig*** does the thing.

Sean throws ***qookies***.

Contract upheld?



p	q	$p \to q$
T	T	T

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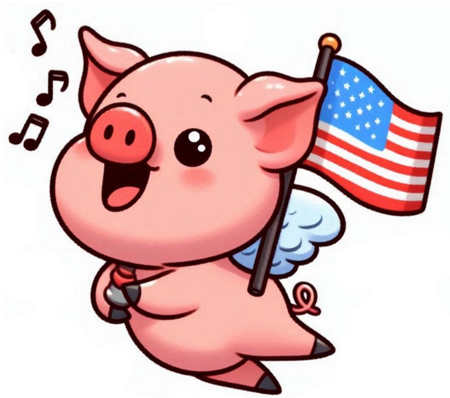

p	q	$p \to q$
F	F	T
T	T	T

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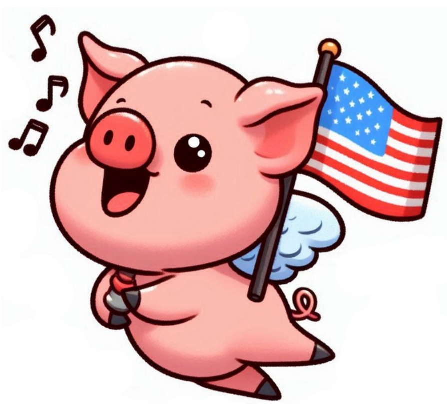

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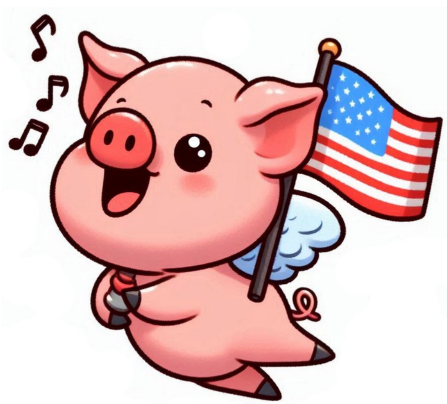

p	q	$p \to q$
F	F	T
F	T	T
T	F	F
T	T	T

A Contract (from Friday):

If a flying pig bursts into the room and sings a pitch-perfect version of the national anthem, then Sean will throw cookies to the class.

p	q	$p \to q$
F	F	T
F	T	T
T	F	F
T	T	T

p	q	$p \to q$
F	F	T
F	T	T
T	F	F
T	T	T

An implication is false only when the antecedent is true and the consequent is false.

Every formula is either true or false, so these other entries have to be true.

p	q	$p \rightarrow q$
F	F	T
F	T	T
T	F	F
T	T	T

Important observation:
The statement $p \rightarrow q$ is true whenever $p \wedge \neg q$ is false.

p	q	$p \to q$
F	F	T
F	T	T
T	F	F
T	T	T

An implication with a false antecedent is called ***vacuously true***.

An implication with a true consequent is called ***trivially true***.

p	q	$p \to q$
F	F	T
F	T	T
T	F	F
T	T	T

Please commit this table to memory. We're going to need it, extensively, over the next couple of weeks.

“If at first you don’t succeed, try again.”

“If at first you don’t succeed, try again.”

FirstSucceed : You succeed at first.

“If at first you don’t succeed, try again.”

FirstSucceed : You succeed at first.

TryAgain : You ought to try again.

“If at first you don’t succeed, try again.”

FirstSucceed : You succeed at first.

TryAgain : You ought to try again.

$$\neg \textit{\textbf{FirstSucceed}} \rightarrow \textit{\textbf{TryAgain}}$$

"If at first you don't succeed, try again."

$\boldsymbol{p}$: You succeed at first.

$\boldsymbol{q}$: You ought to try again.

$$\neg \boldsymbol{FirstSucceed} \rightarrow \boldsymbol{TryAgain}$$

"If at first you don't succeed, try again."

$\boldsymbol{p}$: You succeed at first.

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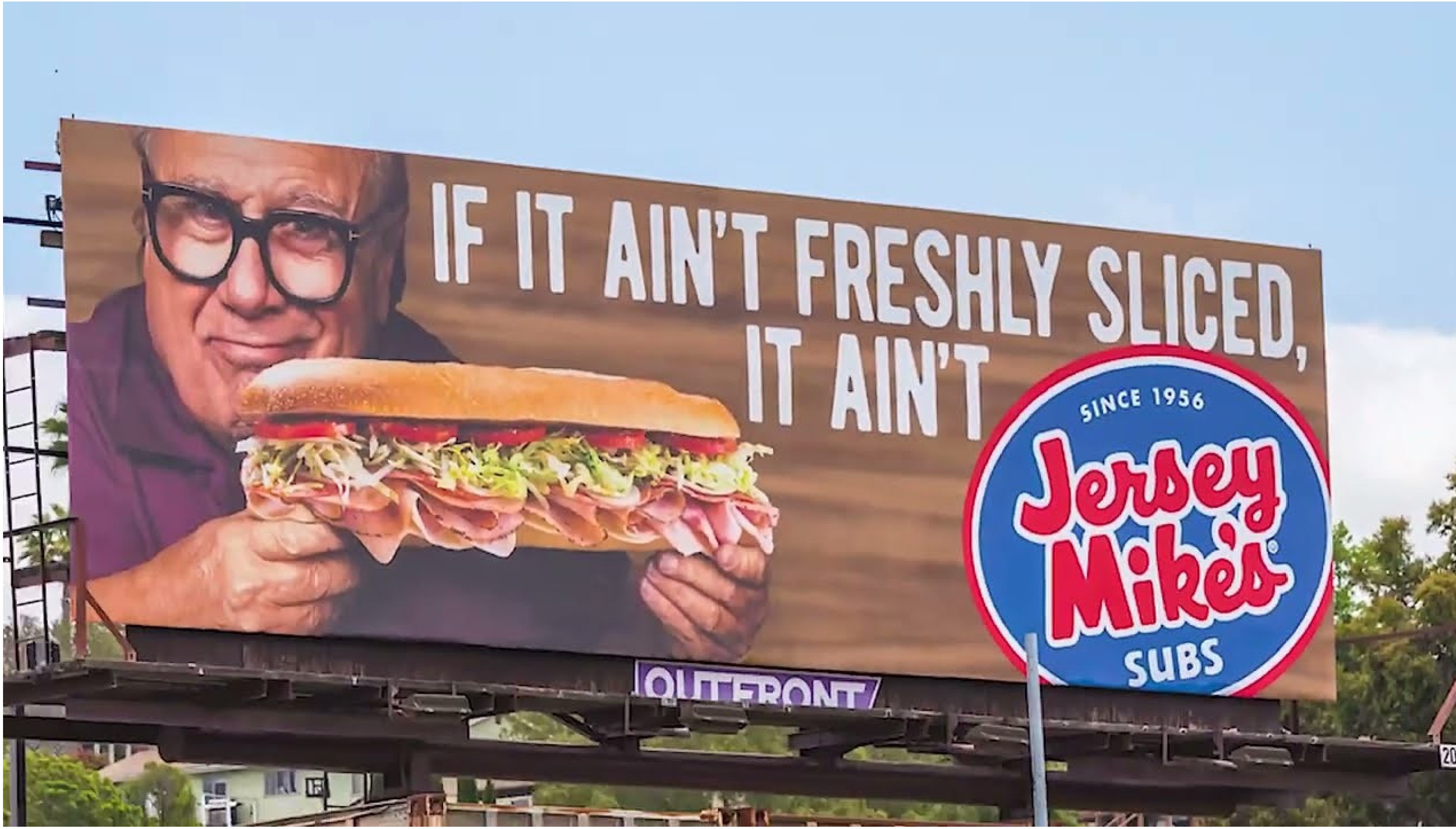
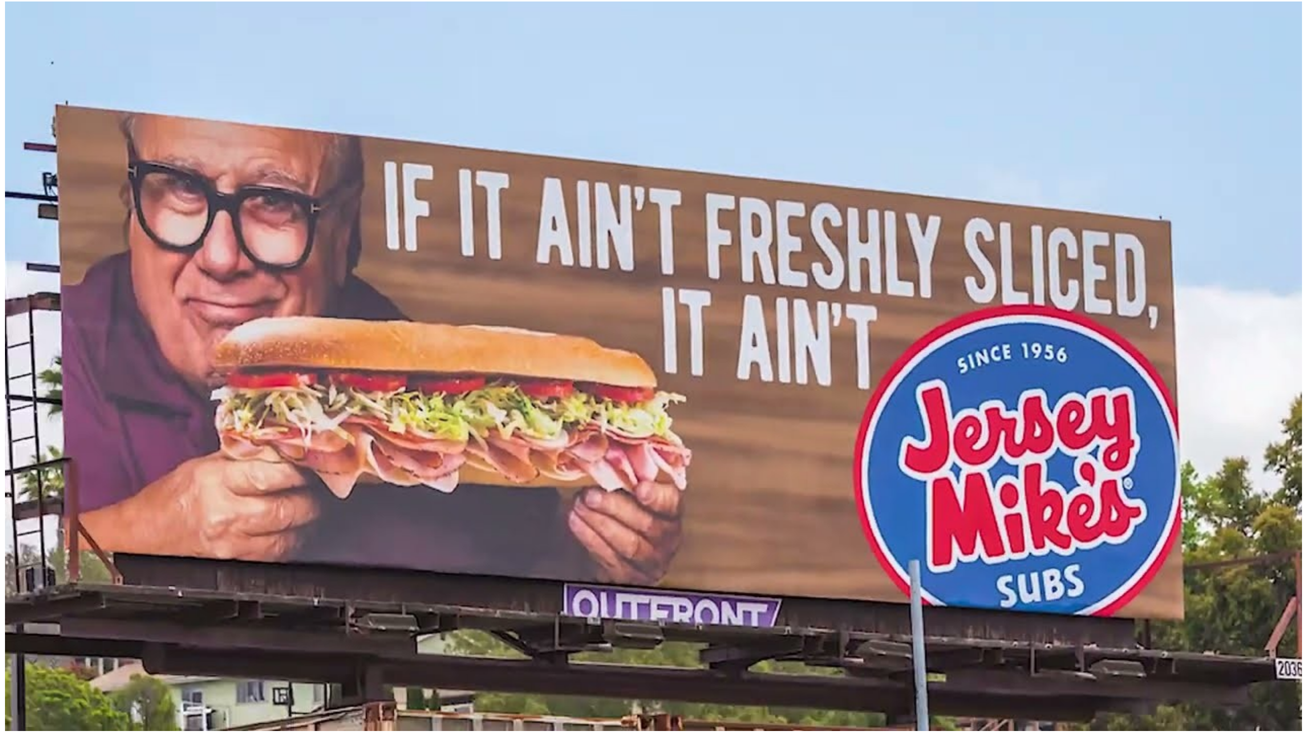
$$\neg \boldsymbol{p} \rightarrow \boldsymbol{q}$$


IF IT AIN'T FRESHLY SLICED,
IT AIN'T
SINCE 1956
Jersey Mike's
SUBS
OUTFRONT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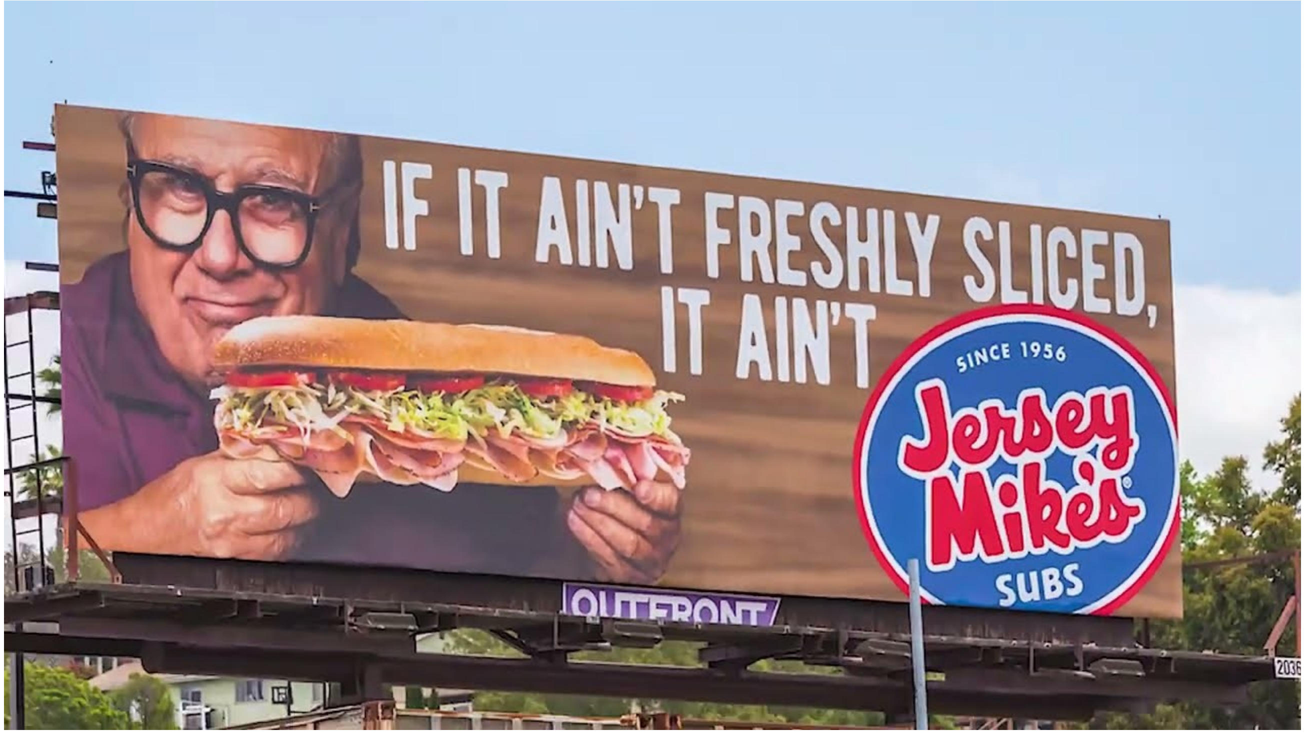
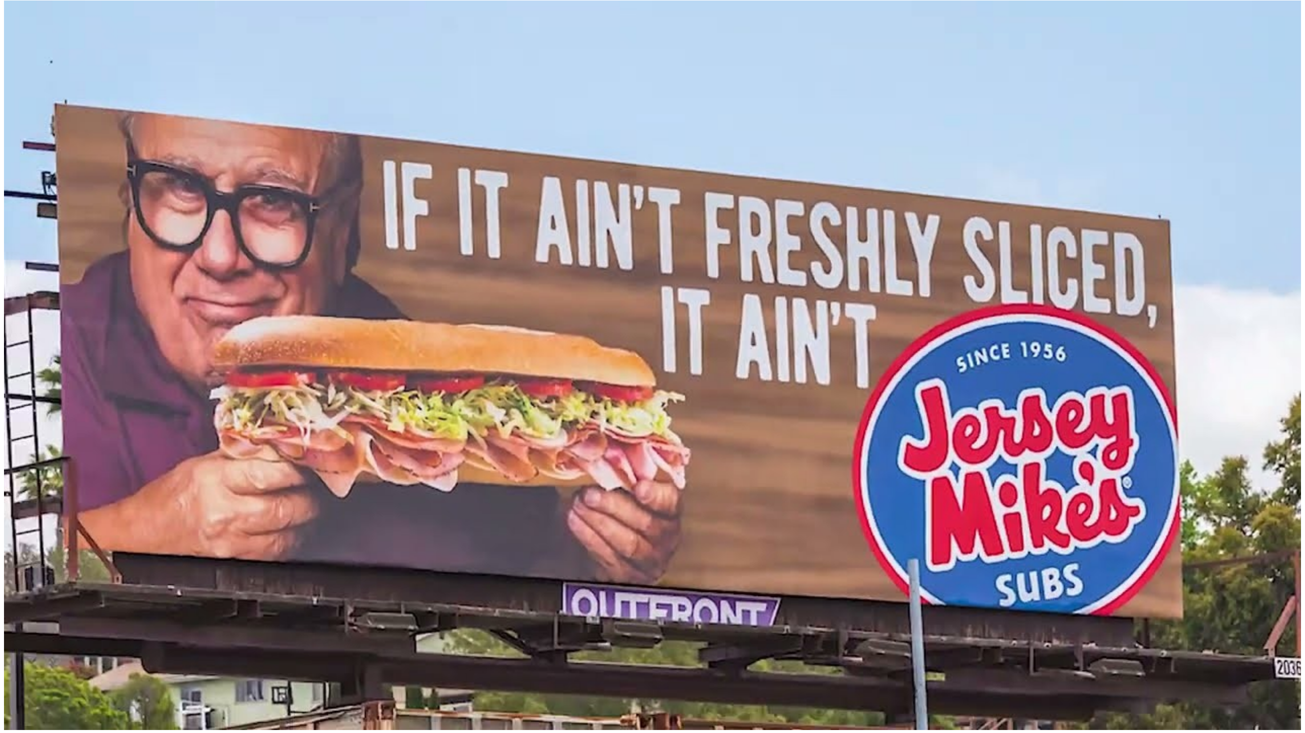
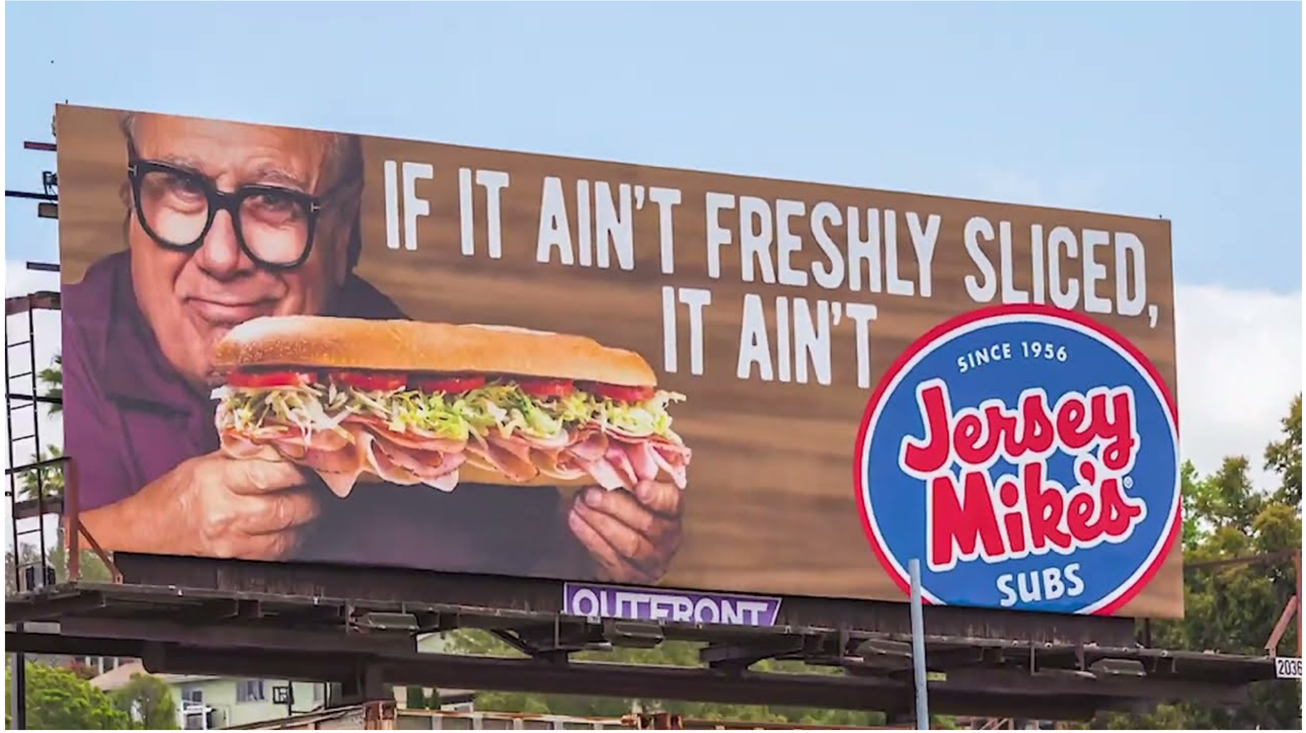


JerseyMikes : It’s Jersey Mike’s.



JerseyMikes : It's Jersey Mike's.

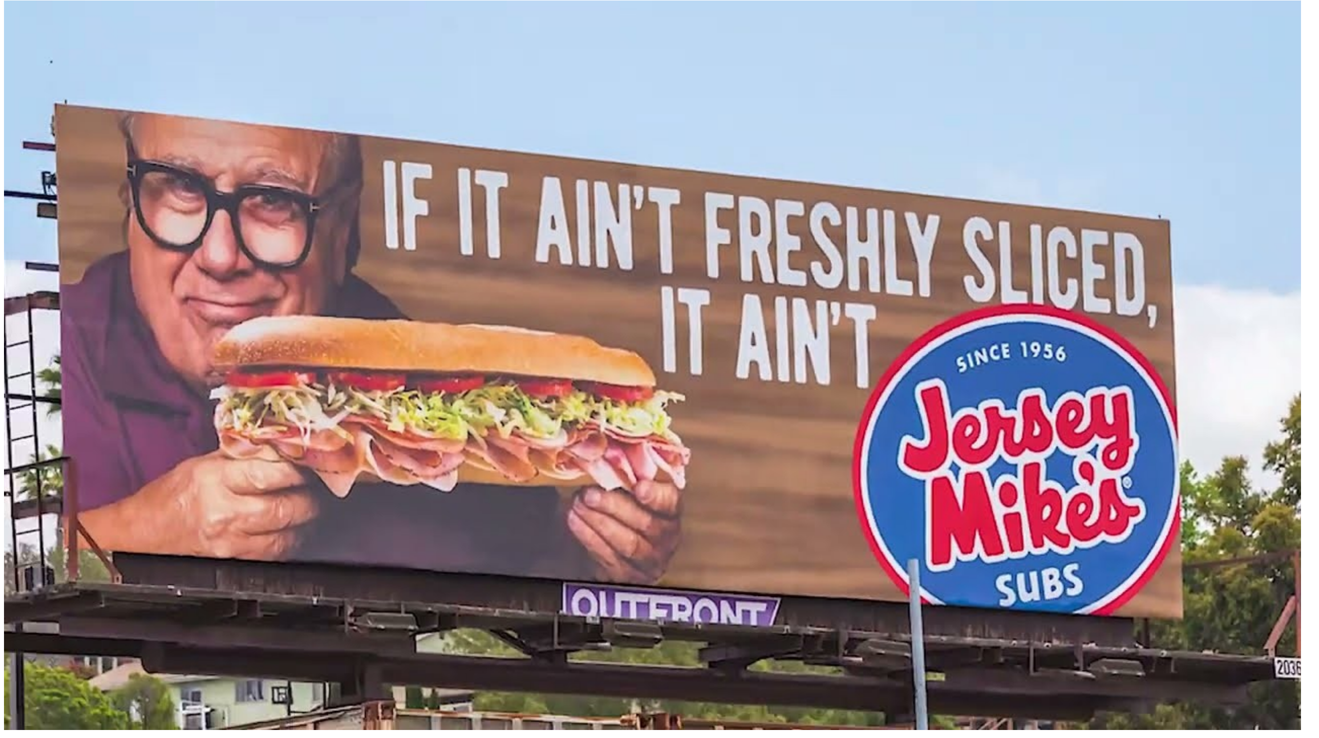
FreshlySliced : It's freshly sliced.



$\textit{JerseyMikes}$: It's Jersey Mike's.

$\textit{FreshlySliced}$: It's freshly sliced.

$$\neg \textit{FreshlySliced} \rightarrow \neg \textit{JerseyMikes}$$



JerseyMikes : It's Jersey Mike's.

FreshlySliced : It's freshly sliced.

$$\neg \textit{FreshlySliced} \rightarrow \neg \textit{JerseyMikes}$$

$$\textit{JerseyMikes} \rightarrow \textit{FreshlySliced}$$

An Important Equivalence

- The truth table for for $p \to q$ is chosen so that the following is true:

$$\boldsymbol{p \to q} \quad \textbf{\textit{is equivalent to}} \quad \boldsymbol{\neg(p \land \neg q)}$$

- Later on, this equivalence will be incredibly useful:

$$\boldsymbol{\neg(p \to q)} \quad \textbf{\textit{is equivalent to}} \quad \boldsymbol{p \land \neg q}$$

Side Note: Contrapositive

We can use truth tables to demonstrate the equivalence of $p \to q$ and $\neg q \to \neg p$.

p	q	$p \to q$	$\neg p$	$\neg q$	
F	F	T	T	T	
F	T	T	T	F	
T	F	F	F	T	
T	T	T	F	F	

Side Note: Contrapositive

We can use truth tables to demonstrate the equivalence of $p \to q$ and $\neg q \to \neg p$.

p	q	$p \to q$	$\neg q$	$\neg p$	$\neg q \to \neg p$
F	F	T	T	T	T
F	T	T	F	T	T
T	F	F	T	F	F
T	T	T	F	F	T

same :)

The Biconditional Connective

The Biconditional Connective

- In our previous lecture, we saw that the statement "p if and only if q" means both that $p \to q$ and $q \to p$.
- We can write this in propositional logic using the ***biconditional*** connective:

$$\boldsymbol{p \leftrightarrow q}$$

- This connective's truth table has the same meaning as "p implies q and q implies p."
- Based on that, what should its truth table look like?

p	q	$p \leftrightarrow q$
F	F	____
F	T	____
T	F	____
T	T	____

How should we fill in these blanks?

Answer at **https://cs103.stanford.edu/pollev**

Biconditionals

- The biconditional connective $p \leftrightarrow q$ has the same truth table as $(p \to q) \land (q \to p)$.
- Here's what that looks like:

p	q	$p \leftrightarrow q$
F	F	T
F	T	F
T	F	F
T	T	T

Biconditionals

- The biconditional connective $p \leftrightarrow q$ has the same truth table as $(p \to q) \land (q \to p)$.
- Here's what that looks like:

p	q	$p \leftrightarrow q$
F	F	T
F	T	F
T	F	F
T	T	T

One interpretation of $\leftrightarrow$ is to think of it as equality: the two propositions must have equal truth values.

Negating a Biconditional

- How do we simplify

  $$\neg(p \leftrightarrow q)$$

  using the tools we've seen so far?
- There are many options, but here are our two favorites:

$$p \leftrightarrow \neg q \qquad\qquad \neg p \leftrightarrow q$$

Question to ponder: what is the truth table for these statements, and where have you seen it before?

Operator Precedence

- How do we parse this statement?

$$\neg x \rightarrow y \vee z \rightarrow x \vee y \wedge z$$

- Operator precedence for propositional logic:

$$\neg$$

$$\wedge$$

$$\vee$$

$$\rightarrow$$

$$\leftrightarrow$$

- All operators are right-associative.
- We can use parentheses to disambiguate.

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- How do we parse this statement?

$$\neg x \rightarrow y \vee z \rightarrow x \vee y \wedge z$$

- Operator precedence for propositional logic:

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$$\rightarrow$$

$$\leftrightarrow$$

- All operators are right-associative.
- We can use parentheses to disambiguate.

Operator Precedence

- How do we parse this statement?

$$(\neg x) \to y \lor z \to x \lor y \land z$$

- Operator precedence for propositional logic:

$$\neg$$

$$\land$$

$$\lor$$

$$\to$$

$$\leftrightarrow$$

- All operators are right-associative.
- We can use parentheses to disambiguate.

Operator Precedence

- How do we parse this statement?

$$(\neg x) \rightarrow y \vee z \rightarrow x \vee y \wedge z$$

- Operator precedence for propositional logic:

$$\neg$$

$$\wedge$$

$$\vee$$

$$\rightarrow$$

$$\leftrightarrow$$

- All operators are right-associative.
- We can use parentheses to disambiguate.

Operator Precedence

- How do we parse this statement?

$$(\neg x) \rightarrow y \vee z \rightarrow x \vee (y \wedge z)$$

- Operator precedence for propositional logic:

$$\neg$$

$$\wedge$$

$$\vee$$

$$\rightarrow$$

$$\leftrightarrow$$

- All operators are right-associative.
- We can use parentheses to disambiguate.

Operator Precedence

- How do we parse this statement?

$$(\neg x) \rightarrow y \vee z \rightarrow x \vee (y \wedge z)$$

- Operator precedence for propositional logic:

$$\neg$$

$$\wedge$$

$$\boldsymbol{\vee}$$

$$\rightarrow$$

$$\leftrightarrow$$

- All operators are right-associative.
- We can use parentheses to disambiguate.

Operator Precedence

- How do we parse this statement?

$$(\neg x) \to (y \lor z) \to (x \lor (y \land z))$$

- Operator precedence for propositional logic:

$$\neg$$
$$\land$$
$$\lor$$
$$\to$$
$$\leftrightarrow$$

- All operators are right-associative.
- We can use parentheses to disambiguate.

Operator Precedence

- How do we parse this statement?

$$(\neg x) \rightarrow (y \vee z) \rightarrow (x \vee (y \wedge z))$$

- Operator precedence for propositional logic:

$$\neg$$
$$\wedge$$
$$\vee$$
$$\rightarrow$$
$$\leftrightarrow$$

- All operators are right-associative.
- We can use parentheses to disambiguate.

Operator Precedence

- How do we parse this statement?

$$(\neg x) \rightarrow ((y \lor z) \rightarrow (x \lor (y \land z)))$$

- Operator precedence for propositional logic:

$$\neg$$

$$\land$$

$$\lor$$

$$\rightarrow$$

$$\leftrightarrow$$

- All operators are right-associative.
- We can use parentheses to disambiguate.

Operator Precedence

- How do we parse this statement?

$$(\neg x) \to ((y \lor z) \to (x \lor (y \land z)))$$

- Operator precedence for propositional logic:

$$\neg$$

$$\land$$

$$\lor$$

$$\to$$

$$\leftrightarrow$$

- All operators are right-associative.
- We can use parentheses to disambiguate.

Operator Precedence

- The main points to remember:
  - $\neg$ binds to whatever immediately follows it.
  - $\wedge$ and $\vee$ bind more tightly than $\to$.
- We will commonly write expressions like $p \wedge q \to r$ without adding parentheses.
- For more complex expressions, we'll try to add parentheses.
- Confused? ***Please ask!***

The Big Table

Connective	Read Aloud As	C++ Version	Fancy Name	Negation
$\neg p$	“not”	`!`	Negation	p
$p \land q$	“and”	`&&`	Conjunction	$\neg p \lor \neg q$ $p \to \neg q$
$p \lor q$	“or”	`||`	Disjunction	$\neg p \land \neg q$
$\top$	“true”	`true`	Truth	$\bot$
$\bot$	“false”	`false`	Falsity	$\top$
$p \to q$	“implies”	*see PS2!*	Implication	$p \land \neg q$
$p \leftrightarrow q$	“if and only if”	*see PS2!*	Biconditional	$p \leftrightarrow \neg q$ $\neg p \leftrightarrow q$

Time-Out for Announcements!

Submitting Work

- All assignments should be submitted through GradeScope.
  - The programming portion of the assignment is submitted separately from the written component.
  - The written component ***must*** be typed; handwritten solutions don't scan well and get mangled in GradeScope.
- All assignments are due at 1:00PM. You have three "late days" you can use throughout the quarter. Each automagically extends assignment deadlines from Friday at 1:00PM to Saturday at 1:00PM; at most one late day can be used per assignment.
  - ***Very good idea:*** Leave at least two hours buffer time for your first assignment submission, just in case something goes wrong.
  - ***Very bad idea:*** Wait until the last minute to submit.
- Your score on the problem sets is the square root of your raw score. So an 81% maps to a 90%, a 50% maps to a 71%, etc. This gives a huge boost even if you need to turn something in that isn't done.

Office Hours

- Office hours have started (as of Sunday)! Think of them as “drop-in help hours” where you can ask questions on problem sets, lecture topics, etc.
  - Check the Guide to Office Hours on the course website for the schedule.
- TA office hours are held in person in the Huang basement. Sean’s are in Durand 331-B.
- Once you arrive, sign up through the CS Office Hours Queue so that we can help people in the order they arrived:

  https://queue.cs.stanford.edu/

- Office hours are *much* less crowded earlier in the week than later. Stop by on Sunday, Monday, and Tuesday!

Back to CS103!

Recap So Far

- A ***propositional variable*** is a variable that is either true or false.
- The ***propositional connectives*** are
  - Negation: $\neg p$
  - Conjunction: $p \land q$
  - Disjunction: $p \lor q$
  - Truth: $\top$
  - Falsity: $\bot$
  - Implication: $p \to q$
  - Biconditional: $p \leftrightarrow q$

Why All This Matters

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- Suppose we want to prove the following statement:

"If $x + y = 16$, then $x \geq 8$ or $y \geq 8$"

Why All This Matters

- Suppose we want to prove the following statement:

“If $x + y = 16$, then $x \geq 8$ or $y \geq 8$”

$$x + y = 16 \to x \geq 8 \lor y \geq 8$$

Why All This Matters

- Suppose we want to prove the following statement:

“If $x + y = 16$, then $x \geq 8$ or $y \geq 8$”

$$x + y = 16 \rightarrow x \geq 8 \lor y \geq 8$$

Why All This Matters

- Suppose we want to prove the following statement:

"If $x + y = 16$, then $x \geq 8$ or $y \geq 8$"

$$\neg(x \geq 8 \lor y \geq 8) \rightarrow \neg(x + y = 16)$$

Why All This Matters

- Suppose we want to prove the following statement:

  “If $x + y = 16$, then $x \geq 8$ or $y \geq 8$”

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“If $x < 8$ and $y < 8$, then $x + y \neq 16$”

Theorem: If $x + y = 16$, then $x \geq 8$ or $y \geq 8$.

Proof: We will prove the contrapositive, namely, that if $x < 8$ and $y < 8$, then $x + y \neq 16$.

Pick x and y where $x < 8$ and $y < 8$. We want to show that $x + y \neq 16$. To see this, note that

$$\begin{aligned} x + y &< 8 + y \\ &< 8 + 8 \\ &= 16. \end{aligned}$$

This means that $x + y < 16$, so $x + y \neq 16$, which is what we needed to show. ■

Why This Matters

- Propositional logic lets us symbolically manipulate statements and theorems.
  - This can help us better understand what a theorem says or what a definition means.
- It's also very useful for proofs by contradiction and contrapositive.
- Being able to negate statements mechanically can reduce the likelihood of taking an negation of contrapositive wrong.

Negation Practice

- Here's a propositional formula that contains some negations. Simplify it as much as possible:

$$\neg(p \wedge q \rightarrow r \vee s)$$

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Next Time

- ***First-Order Logic***
  - Reasoning about groups of objects.
- ***First-Order Translations***
  - Expressing yourself in symbolic math!